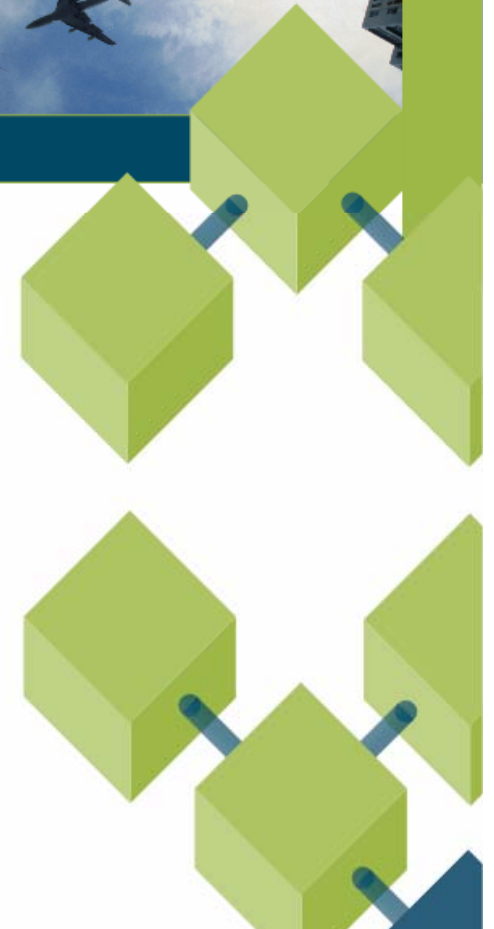ARC Centre for Complex Systems, Australia

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Monte Carlo Simulation in complex systems

Monte Carlo Simulation

- W.S. Gosset in 1908 realised potential of random sampling to generate a random variable

- Von Neumann, Ulam and Metropolis developed algorithms for computer implementation.

- Ulam and Metropolis published the first paper on Monte Carlo in 1949

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Uses of Monte Carlo simulations

There are two main uses:

1. Computing of definite integrals to yield an estimate. E.g. PI, or the Value of a stock option contract

2. Mapping out probability distribution e.g. for risk measurement.

Complex Systems Parameter Space

- **Complex Systems Characteristics**
  - Large number of components
  - Many interactions
  - Large Parameter space
  - Uncertainty in value of parameters
  - Uncertainty in outcomes
    - Stochastic (unpredictable) behaviour

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Examples of Relevant Systems

- Systems with a number of humans interacting through an environment: E.g.
  - Airspace: containing Aircraft, Pilots, Air Traffic Controllers, and airlines.
  - Markets: Contain sellers and buyers.
 - Risk Measurement / Risk management

Portfolio Value (USD MM)

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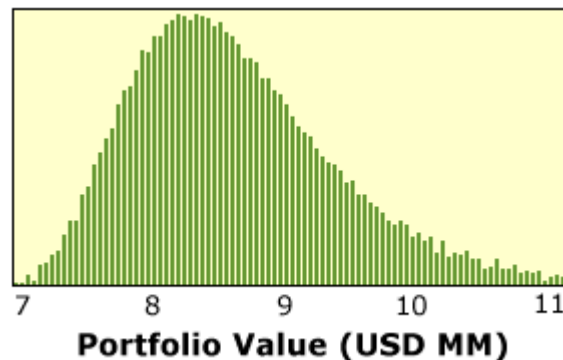
Examples of Relevant Systems

- **Physical systems:**
  - Thermodynamics
  - Quantum Mechanical Systems
  - Quantum Computation
  - Quantum Chemistry
- **Mathematics**
  - Numerical integration, multiple integrals
  - Does not suffer from curse of dimensionality

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- Integration

- Requires convergence

- In 1864 captain O.C. Fox Computed PI dropping Needle over floor boards using Buffon's needle problem: pdf ~ 1/dπ
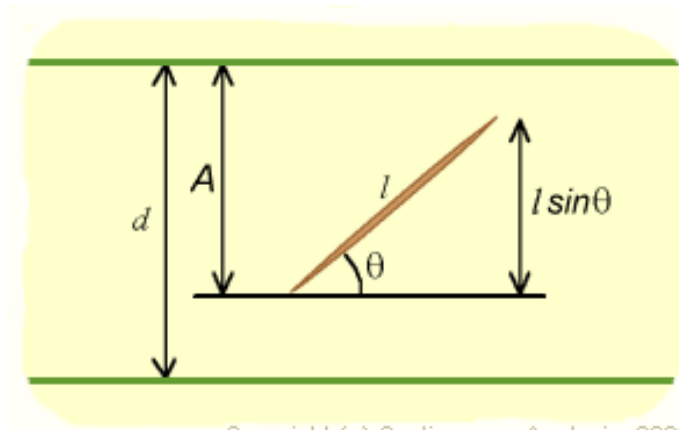
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Parameter Sweep or Scenario Analysis

- Two obvious ways to search the parameter space:
  - Parameter Sweep

    Scan through all possible values of every parameter and simulate/solve system model.

    Problem? Very computationally intensive and slow

  - Scenario Analysis

    Select key parameters and choose several critical values of these parameters and solve.

    Advantages: Much faster and gives a good overview of system behaviour.

    Problem? Statistical significance of outcomes is unknown

Monte Carlo Simulation

- An extension of the scenario approach where key parameters are varied and solution recomputed and recorded.
  - The difference is that the parameter values are randomly chosen from an appropriate probability distribution which is either based on theory or fitted to experimental data.
  - Advantages: Better picture of parameter space with statistically appropriate outcomes. An iterative refinement approach can be used to speed up simulation by fixing parameters to which system is less sensitive.
  - Disadvantages: Slower than scenario analysis

Procedure

1. Specify problem
2. Develop mathematical model
3. Identify key parameters
4. Select probability distributions
   - The potential absence of good data makes this a possible grey area where intuition and experience are as valuable as rigorous analysis. A bit of black magic and taking a risk.
   - But remember the bottom line, any sensible distribution is better than a single valued assumption for a parameter we know is highly variable
   - Financial markets are a classic example:
     - Price of oil when planning to drill well for exploration

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5. Iterate simulation many times while drawing random values of each Monte Carlo variable from distribution:

Pseudo Random number Generation:

This is a whole science/art in itself as it is important to note:

- The outputs of pseudorandom number generators are of course not truly random. Pattern repeat after a time.

- Von Neumann- "Anyone who considers arithmetical methods of producing random numbers is, of course, in a state of sin."

Critical for:

– Cryptography

– Monte Carlo Methods

-....

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Pseudorandom Number Generation (PRNG)

There do exist hardware random number generators. Truly random.

- PRNG Techniques:

  First PRNG – Von Neumann 1946:

  – Take a number- e.g. 1111 (Seed), square it to get: 1234321. Take 4 of the middle digits to get: 3432, this is your first random number. Repeat to get: 7786, and continue.

  – Problem: Sequence repeats too quickly.

  – However much better algorithms now exist: "Mersenne twister" which is fast and has a period of $2^{19937}-1$!

6. Store relevant values of systems variables for each iteration and at each time step.

- This requires some careful management of data as can easily run out of memory. Good computational tools are a must.

- Suggestion: For prototyping and even final model, Matlab is extremely attractive.

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7. Stopping the simulation, two possible criteria:

- Time: Simulation is running over a specified real world period and the final time is reached for the required number of iterations

- Convergence: Either when expected value of Integral converges or sufficient statistics are gathered to map out the probability distribution of the results. Particularly relevant for risk analysis where the tails of the distribution are most relevant but are sampled infrequently.

8. Compute Statistics:

- After simulation has finished, take each system variable of interest and create and array which contains all simulated values of the particular variable. This data can be used to fit a distribution and then calculate moments (mean, std, and others) or the moments can be calculated directly from the data.
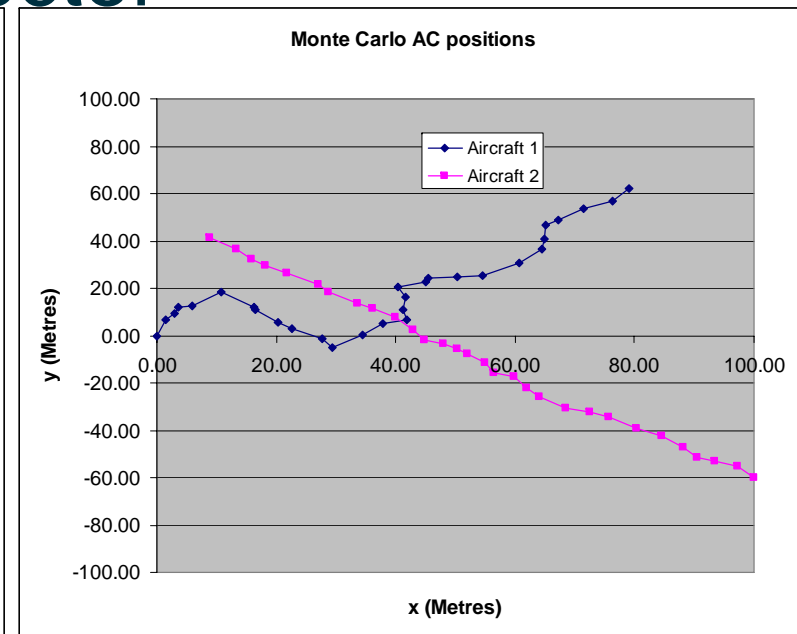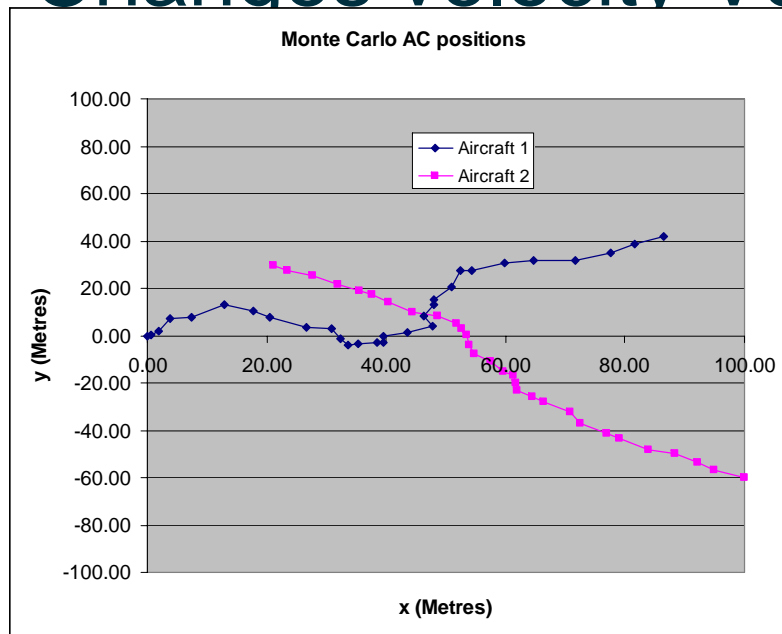
- Introduce noise: Stochastic CA's
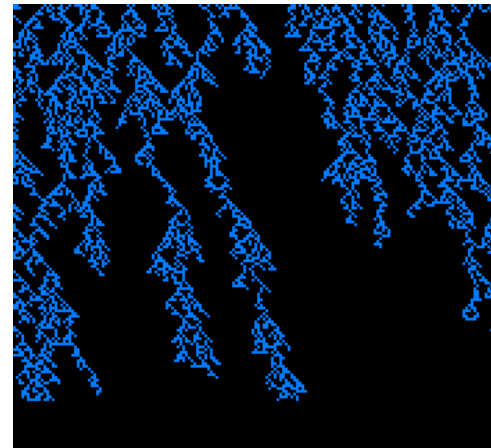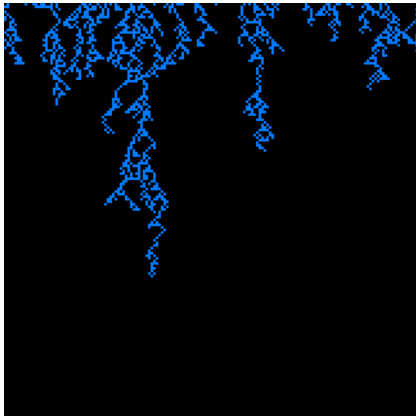
Simple Excel Example: Two Aircraft Collision

- ● Random winds in 2-D
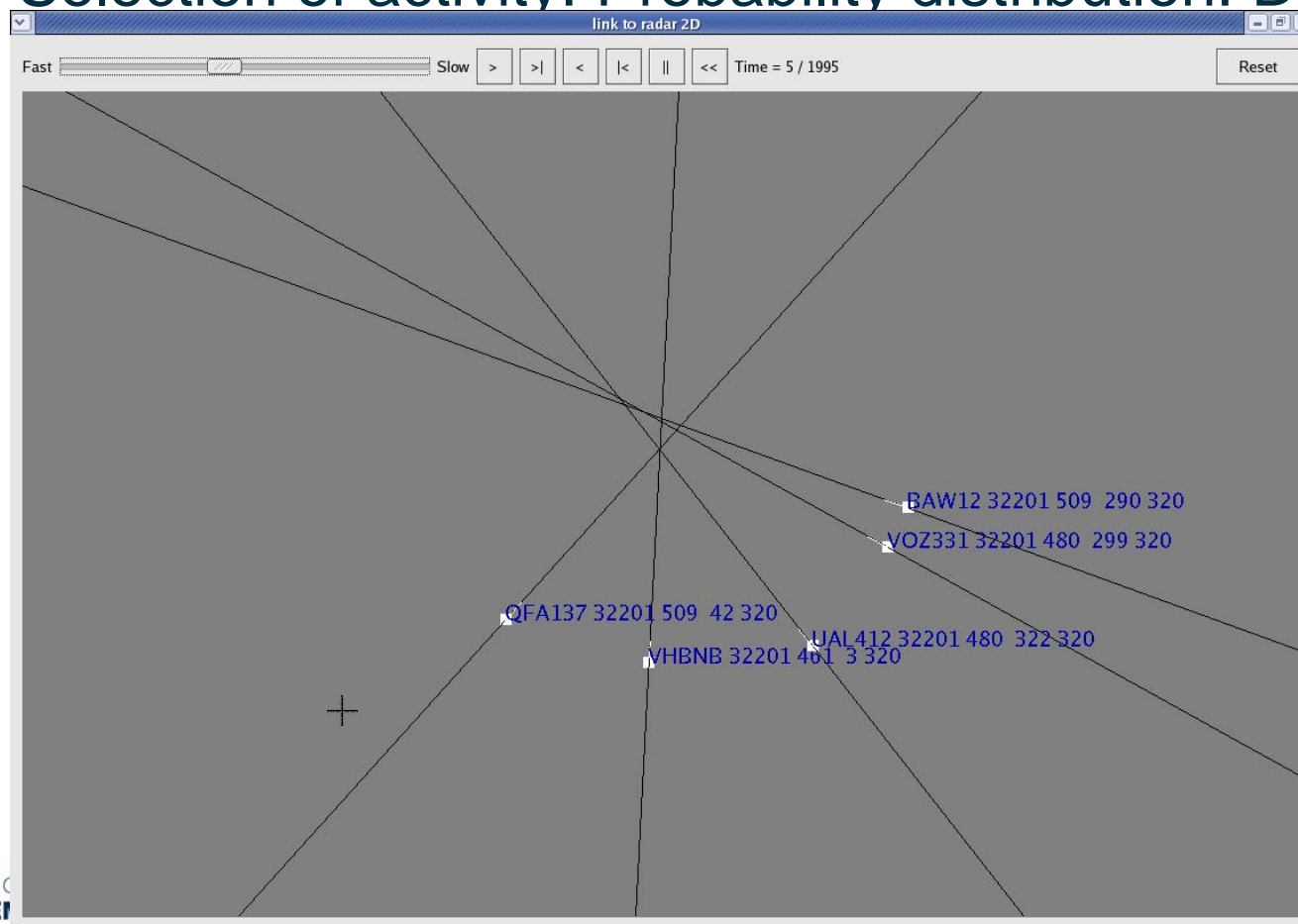- ● Changes velocity Vector

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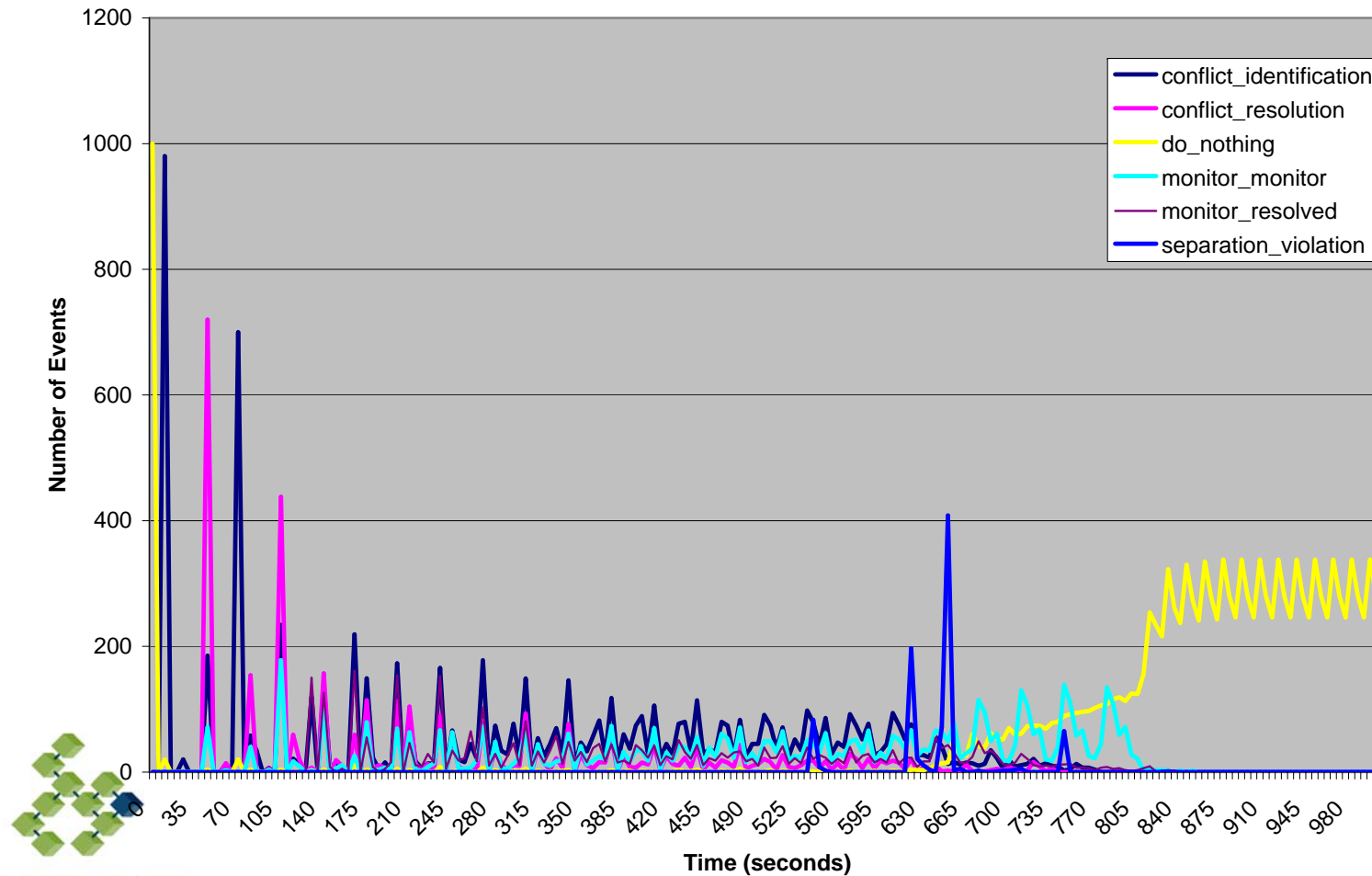**COMPLEX SYSTEMS**

Example simulation

Air Traffic Control Example

- **Prolog Simulation - Key random Variables**
  - Selection of activity: Probability distribution: Discrete

Events

Occurance of Flight Level Changes

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