ARC Centre for Complex Systems, Australia

www.accs.edu.au

How complex is sharing the cost of pollution?

Rodney Beard
Lecturer School of Economics
and ACCS

ARC CENTRE FOR
COMPLEX SYSTEMS

- Basic problem

❑Pollutants such as nutrients and pesticides from agricultural run-off flow down a river system and pollute the coastal marine environment

❑Impact of this on coastal marine economic activities needs to be quantified in terms of the opportunity costs to stakeholders who participate in economic activity that depends on the quality of the coastal marine ecosystem

ARC CENTRE FOR
COMPLEX SYSTEMS

Basic Problem

- Costs to stake holders can be determined either by direct measurement after the damage is done, or by computational modelling of the linkages between coastal agriculture and the marine ecosystem.
- Such models involve for the most part techniques drawing on non-linear dynamics and optimal control theory and are related to questions of ecosystem resilience.

ARC CENTRE FOR COMPLEX SYSTEMS

Non-point source pollution

- This problem is a non-point solution problem in which it is impossible to identify precisely the source of the pollutant
- However, it is possible to share the costs amongst those who are “collectively responsible” for the damage

ARC CENTRE FOR
COMPLEX SYSTEMS

Collective Responsiblity

Joel Feinberg (J. Phil. 1968):

- group liability without fault,
- group liability with noncontributory fault,
- contributory group fault: collective and distributive,
- contributory group fault: collective but not distributive.

ARC CENTRE FOR
COMPLEX SYSTEMS

Fairness and Distributive Justice

- opportunity costs need to be allocated amongst the perpetrators of the e**xternality**
- needs to be done in as fair a manner as possible so that principles of distributive justice are adhered to
- In the case of pollution from coastal agriculture the costs of the pollution need to be shared amongst the farming community along the river.

ARC CENTRE FOR
COMPLEX SYSTEMS

Networks of Agents

- The community of farmers in a river basin may be viewed as a network of agents
- This network is a combination of the physical river network and the **cadastral** land-use network of the farmers
- River basins may be viewed from a complex systems perspective as fractal river networks, characterized by properties such scale-freeness, power law relationships, etc.

ARC CENTRE FOR
COMPLEX SYSTEMS

How do we address the problem of cost-sharing amongst agents located on such networks and how complex is this problem?

- Cost-sharing literature in economics
- Employs game theory as analytic tool
- Can handle networks of agents

ARC CENTRE FOR
COMPLEX SYSTEMS

Game Theory

- The literature on cost-sharing in economics primarily employs cooperative or coalitional form game theory as an analytical tool
- Game theory is a branch of mathematics that is used to study the interaction of multiple agents engaged in economic, social, biological, engineering and physical activity
- As a theory of multi-agent behaviour it is of clear relevance to complex systems science.

ARC CENTRE FOR
COMPLEX SYSTEMS

Game theory

- Previous session non-cooperative game theory
- This session: cooperative Game Theory
- Main differences: communication and ability to commit to strategy

ARC CENTRE FOR
COMPLEX SYSTEMS

Game Theory

- What do we mean by a game?
- Consider a set of *n* agents this is called the **player set**. The potential set of N is denoted by 2^N is the set of all possible coalitions S of N. We can now define what we mean by a **game**.

$$N = \{1, \ldots, n\}$$

ARC CENTRE FOR COMPLEX SYSTEMS

- **Definition 1 (Game)** A cooperative n-person game in coalitional form is an ordered pair (N,v) where and is a map assigning to each coalition S a real number *v*, such that v(0)=0 (Tijs 2003: 60-61).
- The function v is called the characteristic function, the worth or the value of a coalition

$$v : 2^{N} \rightarrow \Re$$

ARC CENTRE FOR
COMPLEX SYSTEMS

- A solution concept determines a division of the “cake” v(S) between the players of a coalition
- Example solution concepts: core, nucleolus, shapley value, kernel, bargaining set

Games on Fixed Networks

- Seminal paper:

R. Myerson (1977) Mathematics of Operations Research

- Also referred to as games with co-operation structures
- Possible coalitions constrained
- Alternative Games with coalition structures originating with G. Owen (1977)

ARC CENTRE FOR
COMPLEX SYSTEMS

Games with Cooperation Structures

- These are games defined on a network or graph
- Seminal paper GRAPHS AND COOPERATION IN GAMES. By: Myerson, Roger B.. ***Mathematics of Operations Research***, Aug77, Vol. 2 Issue 3, p225, 5p;
- A game with cooperation structure is given by the triple

$$\Gamma = (N, v, g)$$

ARC CENTRE FOR
COMPLEX SYSTEMS

Games with Cooperation Structures

- A graph on the set of players N is given by a set of unordered pairs of members of N (Note: ordered pairs would give a directed graph)
- These pairs represent links between players n:m,

$$n, m \in N$$

ARC CENTRE FOR COMPLEX SYSTEMS

Cost Allocation Games

- Municipal cost-sharing problems
- A group of N towns are considering the cost of constructing a shared water treatment facility. Each town has a minimum water requirement that can be met either by themselves or through a water sharing agreement. c(S) is the minimum cost of supplying members of a coalition of towns by the most efficient means.

ARC CENTRE FOR
COMPLEX SYSTEMS

Cost Allocation Games

- Airport Games
- Consider an airport with one runway. There are m different types of aircraft. is the cost of constructing a runway for an aircraft of a given type *k*. N_k is the set of aircraft landings of type k in a given time period. The players of the game are landings of aircraft.

ARC CENTRE FOR COMPLEX SYSTEMS

Airport Games

- Original case study Littlechild and Owen 1973
- Birmingham airport 13,572 landings 11 different aircraft types
- Aircraft landings charges calculated using
- Shapley value and nucleolus

ARC CENTRE FOR
COMPLEX SYSTEMS

Complexity

- How does this relate to complexity?
- There are three broad notions of complexity that can be identified and that can also be related to each other
- The first is **algorithmic complexity**
- The second **informational complexity (Kolmogorov complexity)**
- Thirdly **information based complexity**

ARC CENTRE FOR
COMPLEX SYSTEMS

- Algorithmic complexity is concerned with the computational effort required to solve a particular problem by implementing a given algorithm
- Informational complexity falls into two categories: Kolmogorov complexity which analyzes the informational cost of computation using a Turing machine model of information processing and information based complexity (IBC) which uses a real number model of computation. (see notes for details)

ARC CENTRE FOR
COMPLEX SYSTEMS

Complex systems

- We can now define what we mean by complex systems:

  "Complex systems are systems that generate problems that are computationally intractable".

- Whereby tractability may be defined based on complexity in any of the above senses.

ARC CENTRE FOR
COMPLEX SYSTEMS

Some abbreviations

- wvG –weighted voting game
- wGG weighted graph game
- MBG-minimal base game
- MFG minimum forest game
- MSTG minimum spanning tree game
- Following results taken from Bilbao, Fernandez and Lopez

ARC CENTRE FOR
COMPLEX SYSTEMS

Computational Complexity Results for Some Cooperative Games

Game	Concept	Result
wGG	Core membership	P
MBG,MFG	“	?
wGG	Core empty	P
MFG	Core empty	P

ARC CENTRE FOR
COMPLEX SYSTEMS

More results

Game	Concept	Result
MSTG	nucleolus	NP-hard
MFG	subadditivity	O(size(v))
MBG	submodularity	Oracle-polynomial time
MFG	submodularity	O(size(v))

ARC CENTRE FOR
COMPLEX SYSTEMS

Shapley Value

- Consider the problem of 3 agents sharing joint costs

Example: 3 residents of a sharehouse sharing rent

Who should pay how much rent?

The residents are called A,B,D

ARC CENTRE FOR COMPLEX SYSTEMS

Shapley value

- Each of them pays 200 rent per week to rent by themselves
- If they share a house they pay 350 rent between them
- If A and B or A and D share they can rent for 200
- If B and D rent they pay 350

ARC CENTRE FOR
COMPLEX SYSTEMS

Shapley Value

order	A	B	D
ABD	200	0	150
ADB	200	150	0
DAB	0	150	200
DBA	0	150	200
BDA	0	200	150
BAD	0	200	150
Average	66.67	141.67	141.67

ARC CENTRE FOR
COMPLEX SYSTEMS

Shapley value

- Mathematically

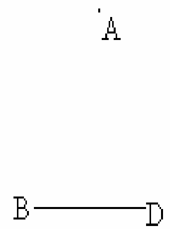
$$x_i = \sum_{s=0}^{n-1} \sum_{S \in A_i(s)} \frac{s!(n-s-1)!}{n!} \{C(S \cup \{i\}) - C\{S\}\}$$

- Encapsulates fairness through equal treatment of equals property
-Not appropriate if agents are postioned on a graph
--coalitions need to be restricted -> Myerson value

ARC CENTRE FOR
COMPLEX SYSTEMS

Myerson Value

- Exercise (pen and paper or spreadsheet): Now assume that B and D are a couple who will not live separately how do the results change?



- A few minutes to work this out

ARC CENTRE FOR
COMPLEX SYSTEMS

Myerson Value

- Solution:

order	A	B	D
ABD	200	200	-50
ADB	200	-50	200
DAB	200	-50	200
DBA	0	150	200
BDA	0	200	150
BAD	200	200	-50
Average	133.3	108.3	108.3

Allocating the cost of pollution

- Consider a river system
- Pollutant flows downriver
- How do we share the costs fairly between players?

ARC CENTRE FOR
COMPLEX SYSTEMS

The Model

- **Players utility is given by a quasi-linear utility function with monetary transfer t**
- **The game is therefore a TU-game**
- ***v(S)* is the characteristic function**

$$U_i\left(x_{ik}, t_{ik}\right) = d_i\left(x_{ik}\right) + t_{ik}$$

ARC CENTRE FOR
COMPLEX SYSTEMS

Computational Algorithm

- **Step 1: To find monetary transfers solve following mathematical programming problem**

$$\min_{\{x,t\}} \sum_{i \in N} U_i\left(x_i\left(N\right), t_i\right)$$

$$s.t.$$

$$\sum_{i \in N} t_i \leq 0$$

$$\sum_{i \in P_j} \left(x_i - e_i\right) \leq 0, \forall j \in N$$

ARC CENTRE FOR
COMPLEX SYSTEMS

Computational Algorithm

- **Step 2: To find c(S)**

$$\min_{\{x\}} \sum_{i \in S} U_i\left(x_i\left(S\right)\right)$$

$$s.t.$$

$$\sum_{i \in P_j} \left(x_i - e_i\right) \leq 0, \forall j \in S$$

ARC CENTRE FOR
COMPLEX SYSTEMS

Computational Algorithm

- Step 3: Evaluate

$$c\left(S\right)=\sum_{i\in S} d_i\left(x_i^{**}\left(S\right)\right)$$

$$v_i^{*}=c\left(P_i\right)-c\left(P_i^{0}\right)$$

ARC CENTRE FOR
COMPLEX SYSTEMS

Example

$$N = \{1, 2, 3, 4, 5\}$$

- Player 1 is downstream and player 5 upstream
- Use a quadratic damage function
- Assume limits to pollution absorption increase as one moves downstream
- Use Excel solver to solve mathematical programming problems and compute the downstream incremental distribution
- What happens if some farmers are on different branches of the network?
- Can you modify the model to allow for this?

ARC CENTRE FOR
COMPLEX SYSTEMS

Evolution and Dynamics

- Dynamic or multiperiod coaltional form games are possible
- Main issue: time consistency
- Time consistency of cooperative game has been discussed extensively in the dynamic games literature with important contributions to the literature
- Petrosjan [18], Filar and Petrosjan [7], Zakharov and
- Dementieva [22].
- However there have only been a few studies that utilize these ideas in an applied context. For example Petrosjan and Zaccour [17] and possibly Tarashnina [21].

ARC CENTRE FOR
COMPLEX SYSTEMS

Evolution of Characteristic Function

t	v(N,t)	v({5,4,3,2},t)	v({5,4,3},t)	v({5,4},t)	v({5},t)
0	36.92901	27.09009	18.69493	11.50769	5.331069
1	36.98019	27.09005307	18.69489	11.50765	5.331032
2	36.98015	27.08760198	18.69244	11.5052	5.328581
3	36.9777	27.08515089	18.6924	11.50516	5.329958
4	36.97908	27.08629052	18.69354	11.5063	5.331097
5	36.98022	27.0874302	18.69468	11.51157	5.336359
6	36.97894	27.0861558	18.69341	11.50888	5.333671
7	36.97626	27.08346764	18.69072	11.50652	5.331314
8	36.9739	27.08111085	18.68836	11.50416	5.328957
9	36.97154	27.07875421	18.68601	11.50181	5.3266

Source: Beard and McDonald (2005)

Some Literature

- E. Algaba et al. On The Complexity Of Computing The Myerson Value By Dividends (http://citeseer.ist.psu.edu/518438.html)
- S. Ambec and Y. Sprumont (2002) Sharing a River, *Journal of Economic Theory* 107: 453-462.
- J. M. Bilbao, J.R. Fernandez and J.J. Lopez Complexity in cooperative game theory, mimeo.
- G. Chartrand and O.R. Oellermann *Applied and Algorithmic Graph Theory*, McGraw-Hill New York 1993.
- U. Faigle et al. On the Complexity of Testing Membership of the Core in Min-Cost Spanning Tree Games, (http://citeseer.ist.psu.edu/faigle94complexity.html)
- M.R. Garey and D.S. Johnson *Computers and Intractability: A Guide to the Theory of NP-Completeness*, San Francisco: W.H. Freeman 1979.
- L.A. Hemaspaandra and M. Ogihara *The Complexity Theory Companion*, Berlin Springer Verlag, 2002.
- T. Ichiishi *Game Theory for Economic Analysis*, New York: Academic Press, 1983.
- M. Li and P. Vitanyi *An Introduction to Kolmogorov Complexity and its Applications*, Berlin: Springer-Verlag 1997.
- R. B. Myerson Graphs and cooperation in games, *Mathematics of Operations Research ,* Vol 2 No. 3 August 1977.
- G. Owen, *Game Theory*, San Diego: Academic Press, 1995.
- G. Owen, *Discrete Mathematics and Game Theory*, Boston, MA: Kluwer, 1999.
- C. Papadimitriou *Computational Complexity*, Reading, MA: Addison-Wesley 1994.
- B.Peleg and P. Sudholter *Introduction to the theory of cooperative games*, Kluwer, 2003.
- A. Schirmer *A Guide to Complexity in Operations Research* (http://halfrunt.bwl.uni-kiel.de/bwlinstitute/Prod/mab/schirmer/research/pdf/wp381.pdf)
- W.W. Sharkey Network Models in Economics, in: M.O. Ball et al. (Eds.) *Handbooks in OR&MS*, Vol. 8. Elsevier 1995.
- S. Tijs *Introduction to Game Theory*, Hindustan book Agency 2003.
- J.F. Traub and A.G. Werschulz *Complexity and information*, Cambridge University Press 1998.
- S. Watanabe *Kolmogorov complexity and computational complexity*, Berlin: Springer-Verlag 1992.

ARC CENTRE FOR
COMPLEX SYSTEMS